<u>CLASE-A-CLASE</u>

Engineering Optimization and Reinforcement Learning

Semestre 2022 – II

Dr. Vadim Azhmyakov, HSE University, Moscow

NO – Nonlinear Optimization;

StO – Stochastic Optimization;

MOO – Multiobjective Optimization

<u>WEEK # 1 (NO)</u>

<u>Introduction to Optimization, Examples, Basic Concepts of the NO, Introduction to Convex Optimization</u>

Some historical remarks related to Optimization Theory and Variational Calculus. Some professional areas of use, introductive examples. Some professional societies in the modern optimization. An example of a real-world optimization in mechanical engineering. The main optimization problem in the abstract form, generic elements of this main problem. First classification of the nonlinear optimization problems (restricted, unrestricted, convex, concave and so on). The generic structure of the modern Optimization: 1. theory, 2. numerical methods, 3. real-world applications. Structure of the course (the existing Micro curriculum).

Concepts of the local and global solutions to an optimization problem, examples. Existence and nonexistence of an optimal solution, multiples minima (multimodal problems), unique optimal solution (unimodal problems). Minima and infimum, examples and counter examples. Definition and examples of a convex function and convex set. Formal properties of the convex sets and formal properties of the convex functions.

Further examples of convex functions and convex sets, practical verification of the basic convexity properties. Convex Optimization: existence of an optimal solution, a local minimum is a global one. Examples of the Convex Optimization problems.

First order characterization of convexity (multidimensional and one dimensional cases). Second order characterization of convexity (multidimensional and one dimensional cases). Examples of applications of the above criteria. Solution of some Convex Optimization problems, discussions.

WEEK # 2 (NO)

Some analytic preliminaries: gradient of vector functions, examples. Stationary points in Convex Optimization. Optimality conditions for the Convex Optimization problems. Simple geometric properties of the optimal solutions in restricted and unrestricted Convex Optimization problems. Examples of applications in simple problems.

The applied statistics / big data practically oriented calculative example of a Convex Optimization problem in linear regression theory. Using of the general optimality conditions for the Convex Optimization problems in this concrete applied example.

WEEK # 3 (NO)

The main Nonconvex Optimization problem with inequalities. The illustration of a general Nonconvex Optimization problem (with mixed inequalities/equalities restrictions). The KKT Theory in the case of a Nonconvex Optimization problem with inequality constraints. Lagrange multipliers.

The KKT Theory in the case of a Convex Optimization problem. The KKT conditions-based solution algorithm. Applications to the nonconvex and convex problems. Examples.

The computational class with the practical exercises of using the KKT conditions. Computational solutions of the further examples of the convex and nonconvex problems based on the KKT framework. The algorithmic approach. A specific case of the Lagrange multipliers equal to zero.

Consideration of some real-world mathematical models and examples in the form of a Nonlinear Optimization problem: MCLP, the security policy optimization problem. Introduction to the Numerical Optimization methods: a concept of a consistent solution method selection. A Python based counter example of an application of a numerical solution method to a simple problem.

An example of a nonregular optimization problem, theoretical and numerical consequences. The Slater´s regularity condition in convex optimization. The concept of a Minimizing Sequence in Numerical Optimization. The conventional Gradient Method. Illustration of the numerical consistency in Numerical Optimization. A simple Python code for the Gradient Method.

WEEK # 4 (NO)

Family of the Lagrange function-based numerical methods. Family of the Gradient Methods (gradient descent, conjugate gradient, projected gradient), examples of the corresponding Python implementation. Advantages and disadvantages. Derivative-free numerical optimization methods: introduction and some examples.

Remark about the second order numerical methods, the SQP numerical approach. Python based implementation of the second order methods.

WEEK # 5 (NO)

Applications of the NO to the Applied Statistics, Big Data, Artificial Intelligence and Machine Learning.

WEEK # 6 (StO)

Introduction to the Stochastic Optimization, Inventory problem. The main problem in Stochastic Optimization. Causes: uncertainties, random nature of variables, parameter variations. Some modelling examples of the Stochastic Programming.

Cases of analytic expressions of the probabilistic constraints (inequalities), the "chance constraints" (confidence levels constraints), the stochastic objective function. Cases of analytic solutions of some problems in Stochastic Optimization. Examples.

WEEK # 7 (StO)

Modelling of the production planning with uncertain demand. Decision Variables and decision criteria. Stochastic Linear - and – Nonlinear Optimization. Numerical Methods in Stochastic Optimization. The Stochastic Gradient Descent approach.

WEEK # 8 (StO)

Introduction to the Monte Carlo Method. Scenario construction, random sampling. Some practically oriented examples – applications of the Monte Carlo Method. General areas of the application of Monte Carlo approach (including Economy, Business, Management, Financial Engineering).

Python based software for the Monte Carlo method. The Method of Recourse problem in Stochastic Optimization. The two-stage problem. Risk optimization. Examples. Some useful bounds and estimations of an optimal solution in Stochastic Optimization.

WEEK # 9 (StO)

Decomposition techniques in Stochastic Optimization. The robust optimization approaches in Stochastic Optimization. The second order numerical methods in Stochastic Optimization. Practically oriented examples – optimization under uncertainties.

Classification and summary of the methods of Stochastic Optimization we considered.

Presentation of some typical examples and topics from StO. Applications of the StO to the Applied Statistics, Big Data, Artificial Intelligence and Machine Learning.

WEEK # 10 (MOO)

Introduction to the Multi-objective (Vector -) Optimization. Some modelling for Multi-objective Optimization. The main problem in the Multi-objective Optimization. Some main definitions and concepts from the Multi-objective Optimization.

The concept of Dominance, the dominance test. The fundamental Pareto definition, the geometrical interpretation of the Pareto optima. Practically oriented examples and models of the Multi-objective Optimization, discussion of the concepts. Multicriterial optimal decision making.

Classical methods in Multi-objective Optimization. The Weighted Sum method. Application of the Weighted Sum method in the convex and non-convex cases. The nested optimization approach – the epsilon-Constraint solution method. Python oriented software and examples.

WEEK # 11 (MOO)

The Weighted Metric method for Multi-objective Optimization. Examples and discussions. Some further solution algorithms for Multi-objective Optimization, a remark about the gradient-based approach. Overview on the modern numerical methods in Multi-objective Optimization.

WEEK # 12 (MOO)

The Multi-Attribute techniques, multi-attribute optimal decision making. Classification of the Multi-Attribute techniques. Applications to the decision support systems.

WEEK # 13 (MOO)

Applications of the MOO to the Applied Statistics, Big Data, Artificial Intelligence and Machine Learning. Presentation of some typical examples and topics from MOO. Python based optimization software for MOO. General remarks about the Course and studied topics.

WEEK # 14 (Some Optimization topics)

Concepts of the Integer / Dynamic / Fuzzy Optimization. Optimization of the networks. Optimization of Dynamic Systems, Optimization methods in Big Data, Financial Engineering, Decision Science.

WEEKS # 15 and 16 (Practical Works)

Definition and discussion about the **Student´s Practical Works.** Problem formulation, problem classification and method selection. Resolution and presentation of the **Student´s Practical Works.**